Are some packings more equal than others?
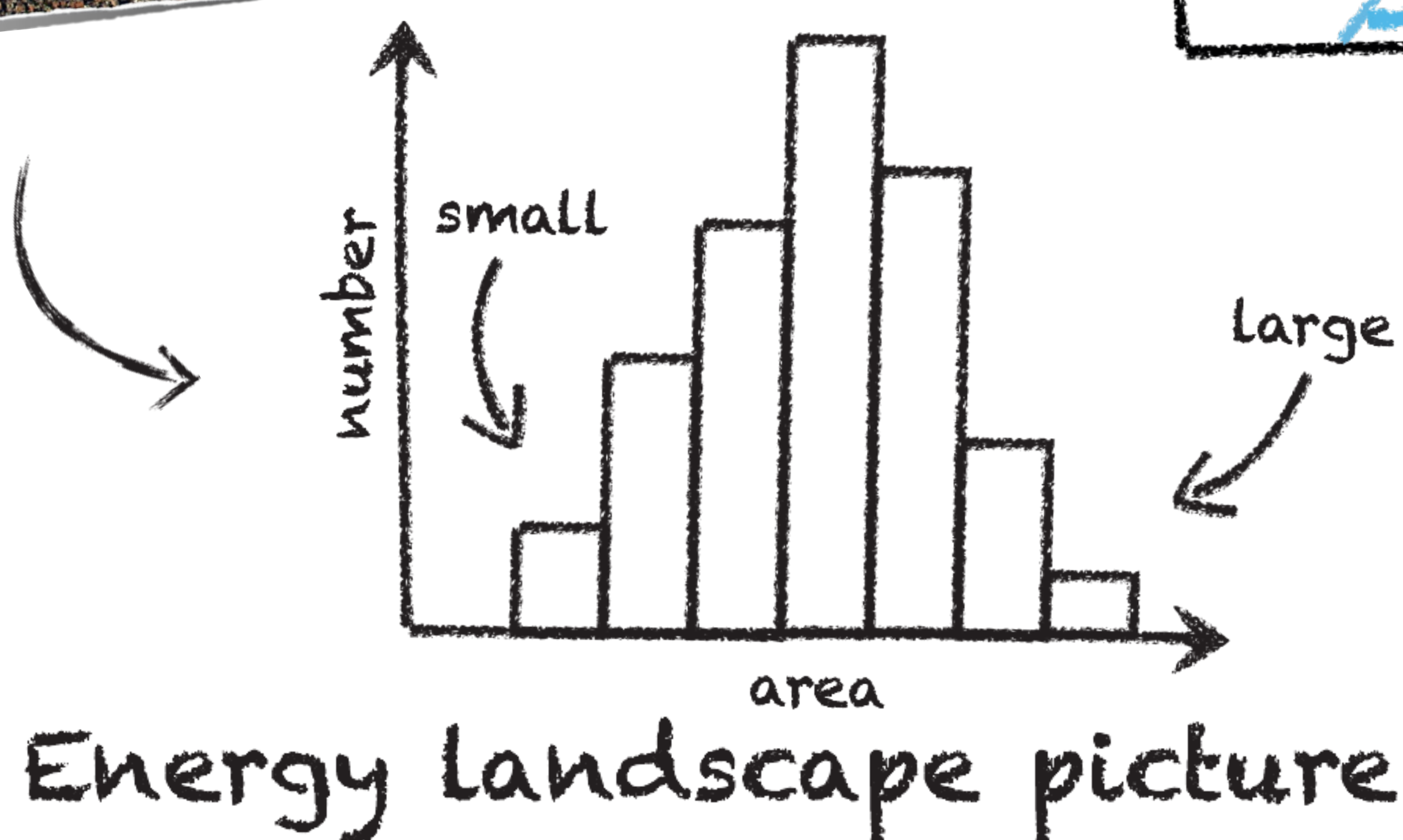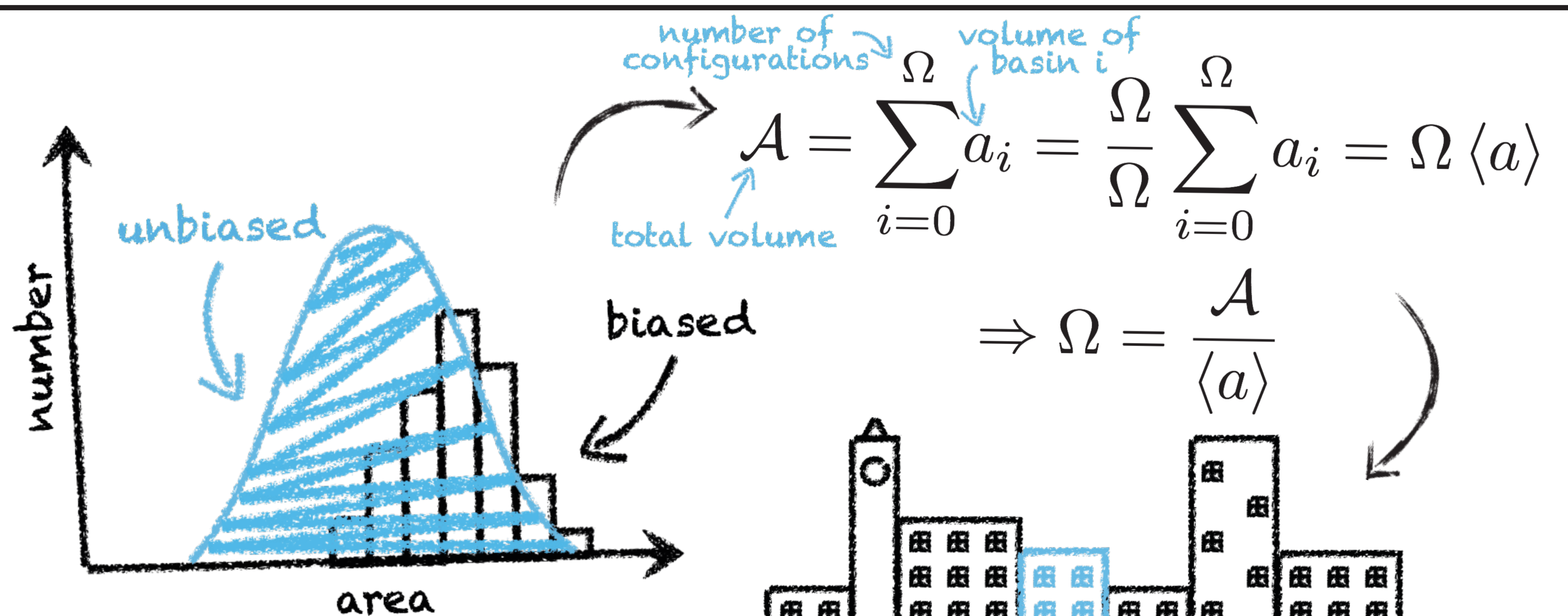## A direct test of the Edwards conjecture
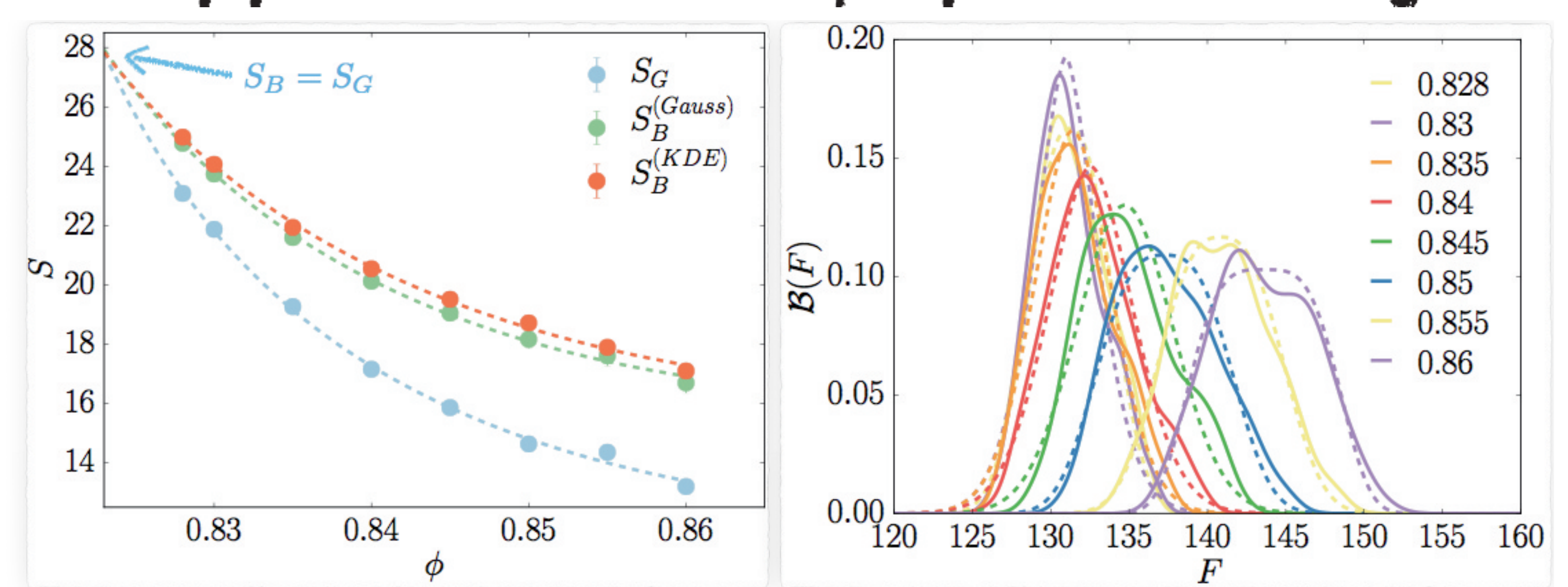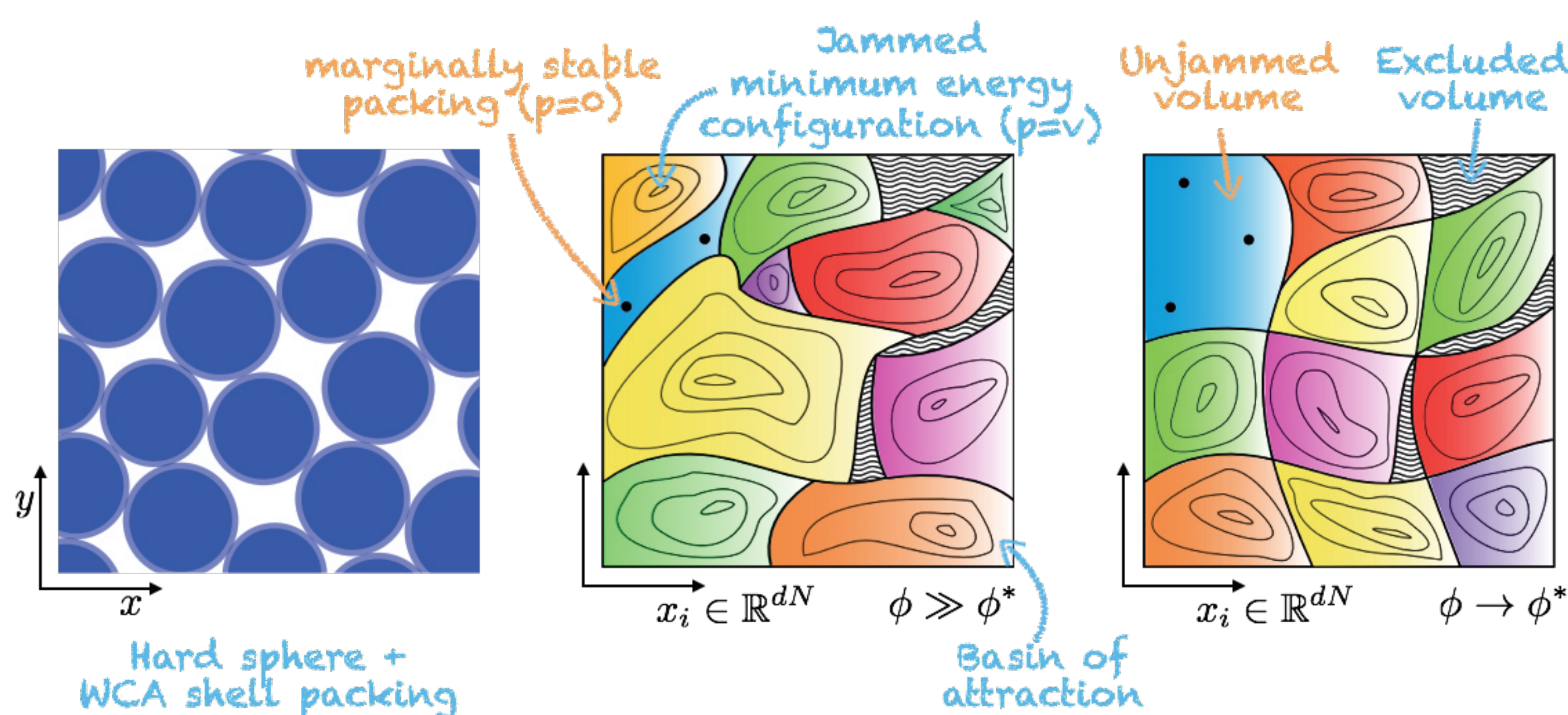
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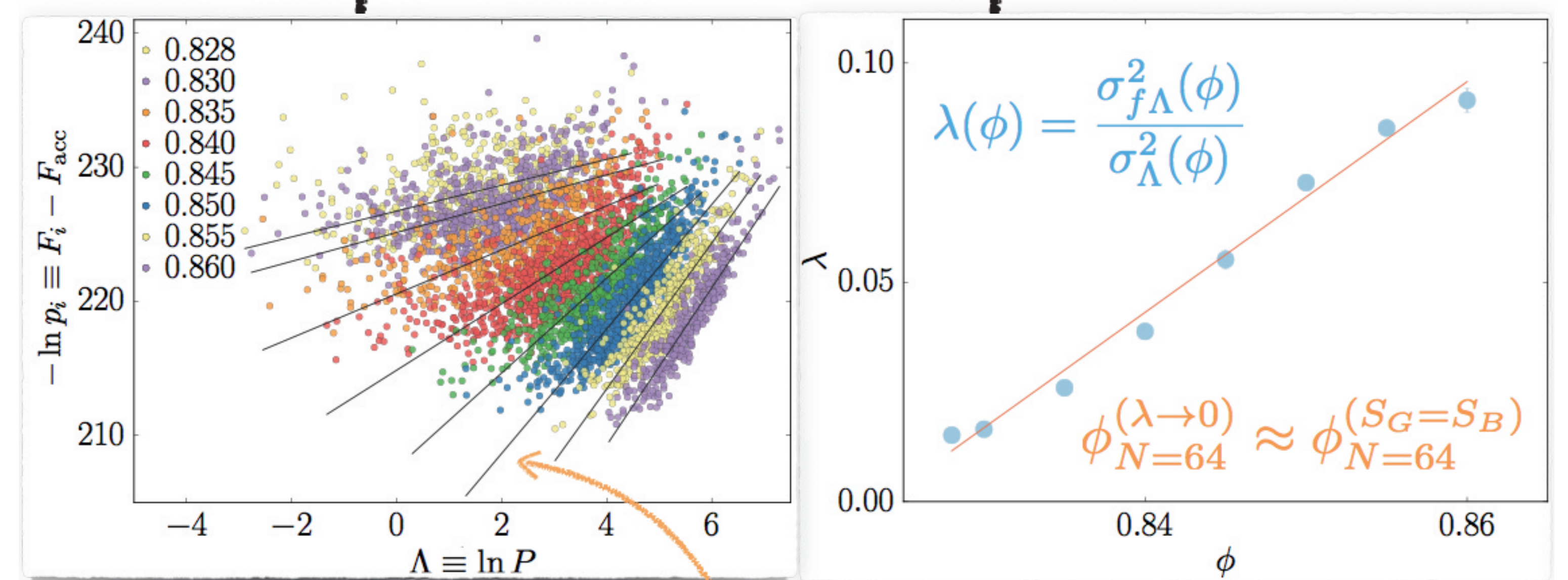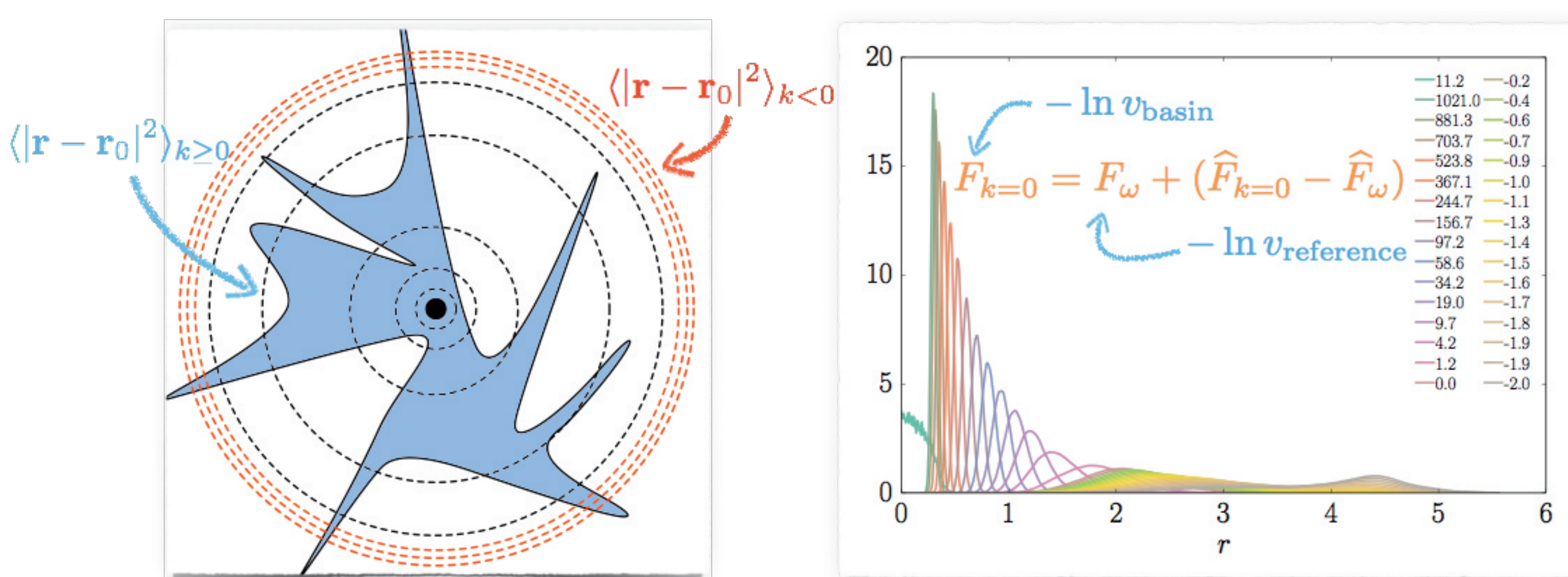
$$\mathcal{A} = \sum_{i=0}^{\Omega} a_i = \frac{\Omega}{\Omega}\sum_{i=0}^{\Omega} a_i = \Omega \langle a \rangle$$

number of configurations — volume of basin i
total volume

$$\Rightarrow \Omega = \frac{\mathcal{A}}{\langle a \rangle}$$

unbiased — biased

Energy landscape picture

marginally stable packing ($p=0$)
Jammed minimum energy configuration ($p=v$)
Unjammed volume — Excluded volume

Hard sphere + WCA shell packing

$x_i \in \mathbb{R}^{dN}$ $\phi \gg \phi^*$ Basin of attraction $x_i \in \mathbb{R}^{dN}$ $\phi \to \phi^*$

Approaching equiprobability

$S_B = S_G$

S_G, $S_B^{(Gauss)}$, $S_B^{(KDE)}$

0.828, 0.83, 0.835, 0.84, 0.845, 0.85, 0.855, 0.86

Measuring basin volumes

$\langle |\mathbf{r} - \mathbf{r}_0|^2 \rangle_{k>0}$
$\langle |\mathbf{r} - \mathbf{r}_0|^2 \rangle_{k<0}$

$-\ln v_{basin}$
$F_{k=0} = F_\omega + (\widehat{F}_{k=0} - \widehat{F}_\omega)$
$-\ln v_{reference}$

Dependence on pressure

$-\ln p_i \equiv F_i - F_{acc}$
$\Lambda \equiv \ln P$

0.828, 0.830, 0.835, 0.840, 0.845, 0.850, 0.855, 0.860

$\lambda(\phi) = \frac{\sigma^2_{f\Lambda}(\phi)}{\sigma^2_\Lambda(\phi)}$

$\phi^{(\lambda \to 0)}_{N=64} \approx \phi^{(S_G = S_B)}_{N=64}$

$\langle f \rangle_{\mathcal{B}}(\phi; \Lambda) = \lambda(\phi)\Delta\Lambda + \langle f \rangle_{\mathcal{B}}(\phi)$

Equiprobability at unjamming

32, 48, 64, 80, 96, 128

$\chi_\Lambda \approx \chi_P$

$\phi^*_{N\to\infty} = \phi^J_{N\to\infty}$

$\lambda(\phi \to \phi^{*/J}_{N\to\infty}) \sim 1/\chi_\Lambda(\phi \to \phi^{*/J}_{N\to\infty}) \to 0$

Extensivity of entropy

$S_G = \langle F \rangle_{\mathcal{B}} - \ln N!$

3D, 2D

$\sum p \ln p$, $\ln\Omega_{KDE}$, $\ln\Omega_G$, $\ln\Omega_{GML}$

entropy is extensive

scale factor B(F)   shape factor B(F)
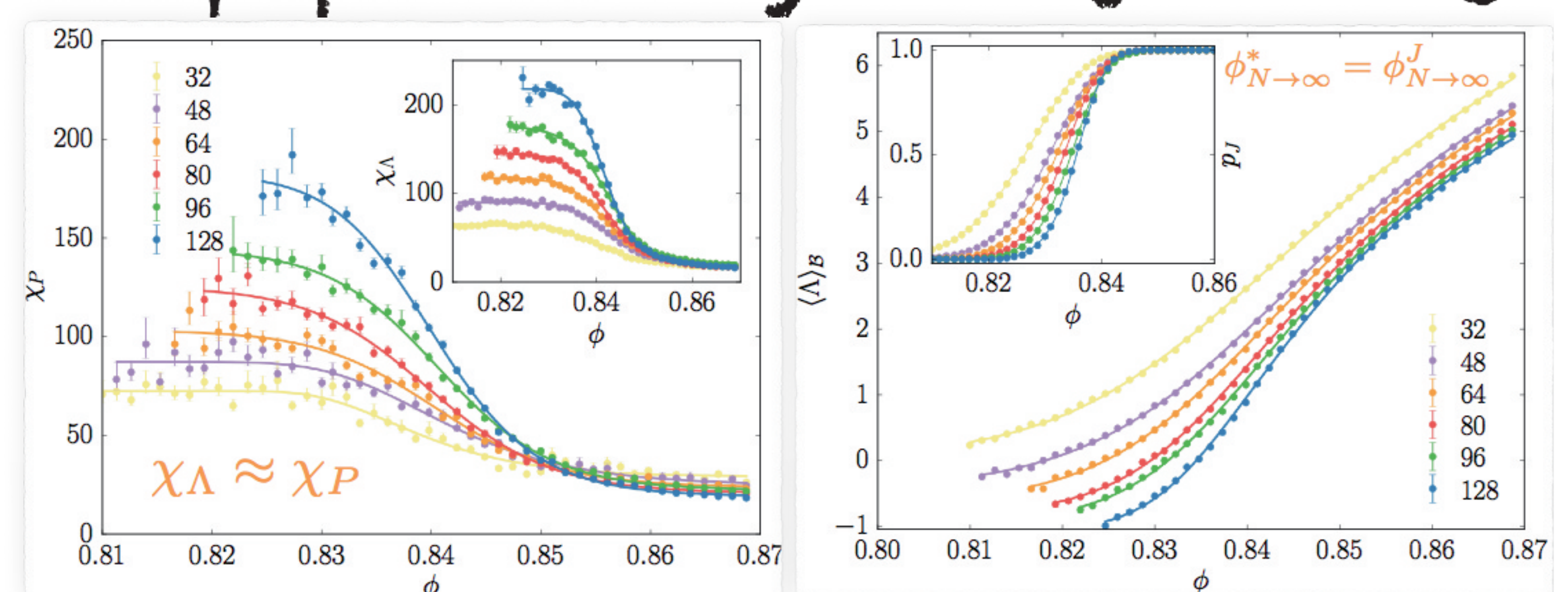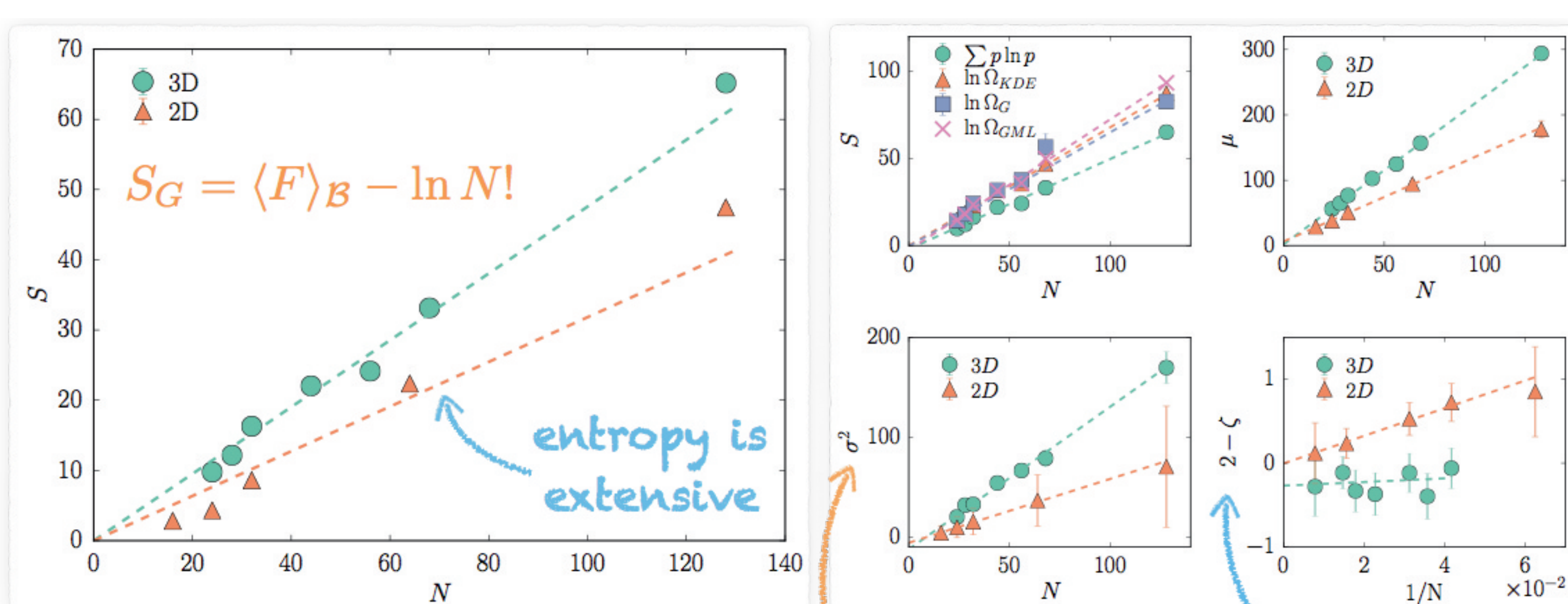
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