Optimal multivalent targeting of cell membranes



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Motivation

A key challenge in biomedical research is the ability to specifically target cells and tissues. Targeting typically relies on identifying a suitable marker, e.g., a highly expressed receptor, and choosing a ligand that strongly and specifically binds to the marker. However, this procedure fails when a suitable marker unique to the targeted cells cannot be identified, notably in many forms of cancer. We show that properly designed multivalent targeting of multiple cognate receptor types results in a specificity toward a chosen receptor density profile, thus demonstrating a general route toward targeting cells without particularly dominant markers.

The challenge How to target cell B in the presence of cells A and C? NP Cell Membrane mobile receptors

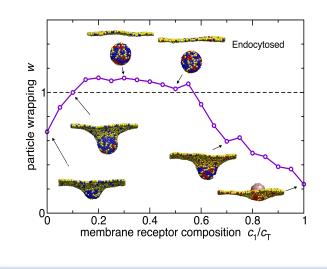
- Each multivalent particle (NP) characterized by the profile of different ligands: ${m p}$
- Membrane is characterized by the receptor concentration vector: c
- Ligands and receptors interact via interaction matrix: ${f K}$

ratio of ligand bound/ unbound probabilities
$$\frac{P_{ij}^{\text{bound}}}{P_i^{\text{free}}} = c_j K_{ij}^A \frac{e^{-\beta \Delta \tilde{G}_i^{enf}}}{h_0} \equiv c_j K_{ij}$$
 ilgand-receptor affinity
$$\frac{\text{ligand-receptor affinity}}{per \text{ligand}}$$

$$f_b(\mathbf{c}, \mathbf{p}, \mathbf{K}) = -\sum_i p_i \ln \left(1 + \sum_i c_j K_{ij}\right)$$

Endocytosis

Particle is only endocytosed when the membrane composition of receptors roughly matched the profile of ligands on the nanoparticle.



Selectivity optimisation

How to specifically target a given receptor composition c^* ?

$$\frac{\partial f_b(\mathbf{c}, \mathbf{p}, \mathbf{K})}{\partial \mathbf{c}}\bigg|_{\mathbf{c}=\mathbf{c}^*} = 0$$

 $S = \det \left(\frac{\mathbf{H}(f_b)}{|f_b|} \right)_{\mathbf{c} = \mathbf{c}^*}$

free energy must be a minimum at c^*

maximize the relative curvature (determinant of the Hessian matrix)

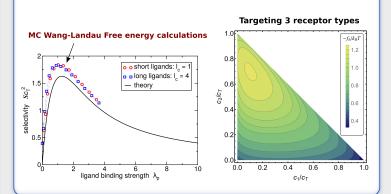
Optimise using Lagrange multipliers

 $\frac{\partial f_b(\mathbf{c}, \mathbf{p}, \mathbf{K})}{\partial \mathbf{p}}\bigg|_{\mathbf{c}=\mathbf{c}^*} = 0.$

robustness condition

SIMPLE ANALYTICAL RESULT: $\lambda_p pprox 1.256 \cdots$

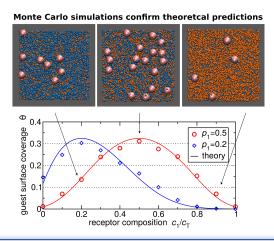
Interaction free energy between each Individual ligand and a targeted membrane should be $\sim 1.3 k_B T$, regardless of the details of the system.



Design Rules

For optimal multivalent targeting of specific receptor composition the following rules apply:

- Individual ligand binding should be weak, each ligand having the probability of being unbound (free): $P_i^{\rm free}=e^{-\lambda_p}\approx 0.3$
- Interaction matrix ${\bf K}$ should be diagonal and inversely proportional to the targeted membrane composition: $K_{ii}\sim 1/c_i$
- Density matching of (cognate) ligands to the targeted receptor composition: $\textbf{\textit{p}} \sim \textbf{\textit{c}}^{*}$



References

T. Curk, J. Dobnikar and D. Frenkel, Optimal multivalent targeting of membranes with many distinct recentors, PNAS 114, 28 (2017)

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